An X-Band Ferroelectric Phase Shifter*

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Summary—**An X-band electrically-tunable ferroelectric phase shifter has been constructed. The phase shifter is reciprocal and consists of a thin ferroelectric slab completely filling the transverse plane of a rectangular waveguide with suitable dielectric matching sections placed symmetrically about the slab forming a band-pass filter. Phase shift is controlled by applying a dc electric field to the ferroelectric. The measured characteristics of this device indicate that incremental phase shifts of 40° to 50° are attainable over a bandwidth of 400 Mc centered about 9.3 kMc with insertion losses ranging from 2 to 6 db. Since the phase shifter does not require a magnetic field for operation, the device can be biased with inexpensive, light-weight equipment requiring negligible dc control power, and the response time can be expected to be fast.**

I. Introduction

Considerable interest has recently been expressed regarding the possibility of using ferroelectric materials for electrically tunable microwave phase shifters. One reason for this interest is that such devices may be utilized in the construction of electrically scanned antenna arrays. The materials presently available for this type of application are ferrites[1] and junction diodes,[2] both of which have certain basic disadvantages. The diode, for example, has the undesirable property that its losses increase with frequency as well as with RF input power; while ferrites have the objectionable feature that they require fairly large amounts of dc drive power to control phase shift.

The purposes of this paper are three-fold. The first is to theoretically predict what can be expected of currently available ferroelectric ceramics. The second is to use these materials in the design and construction of an electrically tunable phase shifter. And the third is to compare theoretical predictions with experimental results.

In the microwave spectrum ferroelectric ceramics behave as isotropic nonlinear dielectrics with both small-signal and large-signal nonlinear properties. The meaning of the small-signal nonlinear behavior, which is of concern here, is that the RF dielectric constant is a function of an applied dc bias field. Among the possible small signal applications of these materials are phase shifting, cavity tuning,[3] tunable filtering, and switching. The salient feature of each of these devices is that they are biased with electrostatic fields which can be established with inexpensive, light-weight equipment requiring neglibible dc drive power. Generally, the response time of such devices can be expected to be fast, typically better than a microsecond.

The phase shifter described here consists of a ferroelectric slab completely filling the transverse plane of a rectangular waveguide, with suitable dielectric matching sections placed symmetrically about the slab, thus forming a band-pass filter. This type of structure is a simple realization of a reciprocal electrically controlled ferroelectric phase shifter; it was chosen as a method of studying the feasibility for using ferroelectrics in this type of device. Among the advantages of this configuration are: high power capabilities; simplicity of construction and effectiveness of matching; ability to describe the device analytically, thereby readily permitting a comparison between theory and experiment; and ease of providing bias to control the phase shift. Its disadvantages are common to all small-signal ferroelectric devices. These are: the necessity of providing a means of temperature stabilization, and the necessity of providing a scheme for the prevention of dc arcing across the ceramic interface at high-bias fields.

II. Small Signal Properties of Ferroelectric Ceramics

For microwave applications the nonlinear properties of ferroelectric ceramics are best described by giving the variation of the complex dielectric constant defined by

$$\epsilon = \epsilon_0(\kappa' - j\kappa'') = \epsilon_0\kappa'(1 - j \tan \delta)$$

as a function of dc bias, with temperature as a parameter. Figs. 1 and 2 give curves of κ' and $\tan \delta$ as a function of dc bias, with temperature as a parameter, for a ceramic composed of 73 per cent barium titanate and 27 per cent strontium titanate. These curves are the results of X-band measurements made at the Stanford Microwave Laboratory, using a transmission technique.[4] They show that the real part of the dielectric constant is ex-

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[1] F. Reggia and E. G. Spencer, "A new technique in ferrite phase shifting for beam scanning of microwave antennas," Proc. IRE, vol. 45, pp. 1510–1517; November, 1957.

[2] R. H. Hardin, E. J. Downey and J. Munushian, "Electrically variable phase shifters utilizing variable capacitance diodes," Proc. IRE, vol. 48, pp. 944–945; May, 1960.

[3] W. J. Gemulla and R. D. Hall, "Ferroelectrics at microwave frequencies," *Microwave J.*, vol. 3, pp. 47–51; February, 1960.

[4] D. A. Johnson, "Microwave Properties of Ceramic Nonlinear Dielectrics," Microwave Labs., Stanford University, Stanford, Calif., Rept. No. 825; July, 1961.

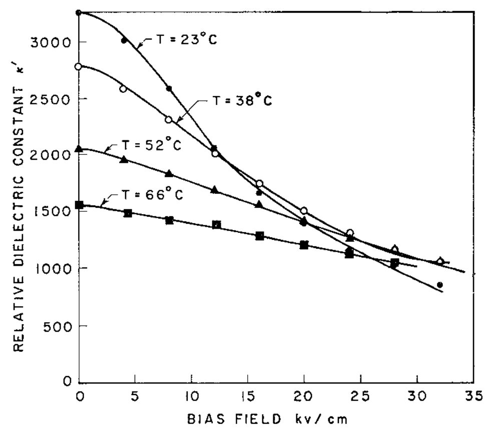

Fig. 1—Relative dielectric constant κ' as a function of dc bias for various temperatures for ceramic of 73 per cent $BaTiO_3$–27 per cent $SrTiO_3$.

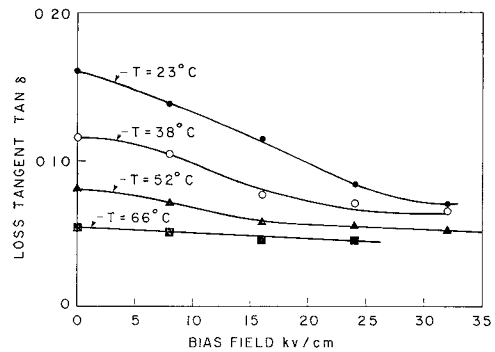

Fig. 2—Loss tangent tan δ as a function of dc bias for various temperatures for ceramic of 73 per cent $BaTiO_3$–27 per cent $SrTiO_3$.

tremely high, ranging between 1000 and 3000, and that the loss tangent or Q of these materials, where $Q=1/\tan\delta$, is moderately high. These results are in general agreement with those obtained by other investigators.[5–8]

For design purposes, Figs. 1 and 2 show that a 50 per cent change in κ' is easily achieved with moderate bias fields and that the material Q ranges between 7 and 20. Since the properties of these ceramics show considerable variation with temperature in the range 23°C to 52°C with the temperature sensitivity decreasing at higher temperatures, it is generally desirable to operate the materials above 40°C, which is a comparatively low-loss region having appreciable nonlinearity.

[5] H. J. Schmitt, "Dielectric constant of barium titanate at 10 kMc," *Z. angew. Physik*, vol. 9, pp. 107–111; March, 1957.

[6] L. Davis and L. G. Rubin, "Some dielectric properties of barium-strontium titanate ceramics at 3000 megacycles," *J. Appl. Phys.*, vol. 24, pp. 1194–1197; September, 1953.

[7] J. G. Powles and W. Jackson, "The measurement of the dielectric properties of high-permittivity materials at centimetre wavelengths," *Proc. IEE* (London), vol. 96, pt. 3, pp. 383–389; September, 1949.

[8] C. B. Sharpe and C. G. Brochus, "Investigation of Microwave Properties of Ferroelectric Materials (final rept.)," University of Michigan Res. Inst., Ann Arbor, Mich., Rept. No. 2732-4-F; March, 1959.

III. Theory and Design of Phase Shifter

A. General Theory

Electrically controlled phase shift is usually achieved by varying the properties of a nonlinear medium in a controllable manner, and can be accomplished by either changing the propagation constant in a propagating circuit or the phase of the reflection coefficient in a reflecting circuit. The former method is employed here in the construction of a reciprocal phase shifter. The problem of finding the change in propagation constant, for a prescribed change in dielectric constant, in inhomogeneously filled waveguides or in more general configurations, is usually difficult or impossible to solve exactly. In such cases, where solutions in closed form cannot be obtained easily, perturbation or variational techniques may be used. It is shown in the Appendix that the incremental behavior of an arbitrary phase shifter can be described by

$$\Delta\phi = 0.115 \frac{\Delta\kappa'}{\kappa' \tan\delta} L \text{ rad}, \tag{1}$$

where

$\Delta\phi$ is the incremental phase shift,
$\Delta\kappa'/\kappa'$ is the fractional change in the real part of the dielectric constant of the ferroelectric, and
L is the insertion loss of the device in db.

In the derivation of (1) it is assumed that the device is well matched, that wall losses are negligible, and that the insertion loss is small (*i.e.*, less than 3 db). The significance of this formula is that it is independent of the configuration of the transmission system and of the shape and location of the ferroelectric.

The primary value of (1) is that for a given insertion loss one can estimate the incremental phase shift (or change in total phase shift) in terms of the properties of the ferroelectric material. As an example, suppose that the insertion loss of a phase shifter was limited to $L=2$ db, and that the loss tangent of the material employed was $\tan\delta=0.05$. For a fractional change in the real part of the dielectric constant of $\Delta\kappa'/\kappa'=0.05$ the incremental phase shift, computed from (1), is given by $\Delta\phi\approx 13°$. This calculation is independent of the configuration of the phase shifter and applies to any geometry having the given variables.

B. Phase Shifter Design

A longitudinal section of the X-band transmission-type phase shifter is shown in Fig. 3. The phase shifter consists of a ferroelectric slab completely filling the cross section of a rectangular guide with low-loss dielectric matching sections placed symmetrically about the slab forming a band-pass filter. Assuming that only the dominant mode propagates, the boundary value problem for such a configuration can be solved exactly,

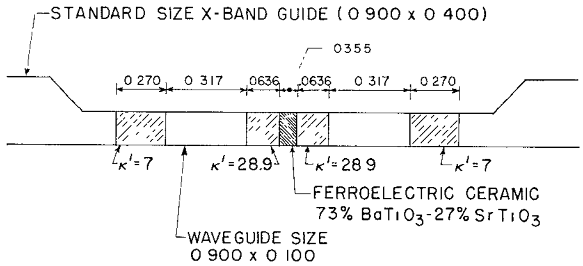

Fig. 3—Longitudinal section of phase shifter. The dielectrics completely fill the transverse plane of the guide.

thereby permitting a complete description of its properties.

By controlling the permittivity of the slab with an external dc bias, both the complex propagation constant and characteristic impedance of the ferroelectric section are varied. Phase shift results from the change in the imaginary part of the propagation constant. Referring to Fig. 3, it can be seen that each dielectric interface or junction presents a discontinuity to an impinging wave. At each of these interfaces the boundary conditions are satisfied by an incident, reflected, and transmitted wave all in the same mode. Using the fact that each waveguide normal mode can be represented by a transmission line of characteristic impedance equal to the wave impedance,[9] the over-all equivalent circuit of the device becomes a series of transmission lines in cascade.

Employing this approach, the properties of the phase shifter can be obtained with a minimum of effort. By permitting only the TE_{10} mode or the dominant mode of the external waveguide circuitry to propagate within the filter, the propagation constant γ and the wave impedance or characteristic impedance defined by $j\omega\mu_0/\gamma$ can be determined for each waveguide section by[10]

$$\gamma^2 = \left(\frac{\pi}{a}\right)^2 - \frac{\omega^2}{c^2}\kappa'(1 - j\tan\delta) \tag{2}$$

and

$$Z_0 = j\omega\mu_0\left[\left(\frac{\pi}{a}\right)^2 - \frac{\omega^2}{c^2}\kappa'(1 - j\tan\delta)\right]^{-1/2}, \tag{3}$$

where γ and Z_0 are, respectively, the propagation constant and characteristic impedance of each of the equivalent transmission lines of the filter and a is the guide width. In the ferroelectric section where $\kappa' > 1000$ and $\tan\delta < 0.1$, these equations can be simplified to

$$\gamma \approx \frac{\omega}{c}\sqrt{\kappa'}\left(\tfrac{1}{2}\tan\delta + j\right), \tag{4}$$

and

$$Z_0 \approx \frac{377}{\sqrt{\kappa'}}. \tag{5}$$

Eqs. (2) through (5), together with Figs. 1 and 2, form the bias for designing the phase shifter. Assuming lossless matching sections and a perfectly matched system, the insertion loss of the device is the intrinsic loss of the ferroelectric section. The insertion loss L and the total phase shift or phase delay ϕ in traversing the ceramic slab are given by

$$L = 4.343\frac{\omega}{c}l\sqrt{\kappa'}\tan\delta \text{ db}, \tag{6}$$

and

$$\phi = \frac{\omega}{c}l\sqrt{\kappa'} \text{ rad}, \tag{7}$$

where l is the length of the ferroelectric slab, and where κ' and $\tan\delta$ refer to the properties of the ferroelectric. The equations for the insertion loss and the phase shift are obtained from the real and imaginary parts of (4).

The incremental phase shift resulting from changes in the permittivity of the ferroelectric is obtained from (7) by evaluating the total phase shift for two different bias fields and subtracting the results. For a fixed temperature the values of κ' used in this calculation are determined from Fig. 1. In this way it is possible to predict how the incremental phase shift through the device varies with bias. Similarly, the behavior of the insertion loss can be obtained from (6) and the curves given in Figs. 1 and 2. It should be noted that, for small changes in the dielectric constant, (6) and (7) can be combined to give (1). Thus the solution of the boundary value problem for the geometrical configuration of Fig. 3 agrees exactly with the general theory, as expected.

Matching into the ferroelectric is achieved by placing low-loss dielectric-impedance matching sections on each side of the ferroelectric slab in a symmetrical manner. The combination of the matching sections and the ferroelectric section thereby unite to form a band-pass filter. Since the ferroelectric is an integral part of the filter and since its properties are controlled by an external dc bias, the filter characteristics depend upon the bias voltage. The filter design is therefore carried out at one bias voltage (the average bias) and at some fixed temperature (chosen here to be 52°C). The dielectric constants and lengths of the impedance matching sections, which can be determined from (2) and (3), are selected so that the system is perfectly matched at two frequencies f_1 and f_2. Thus, at f_1, the lower frequency, the dielectric section adjacent to the ferroelectric is designed to be a quarter-wave matching transformer while the other dielectric section, furthest removed from the ferroelectric, is chosen to be a half-wave line at this

[9] R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 209–214; 1960.
[10] *Ibid.*, p. 182.

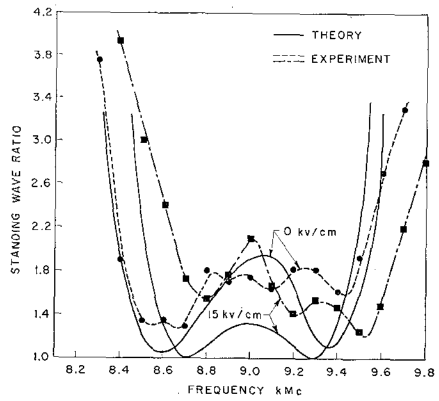

Fig. 4—Measured VSWR characteristics and theoretical predictions.

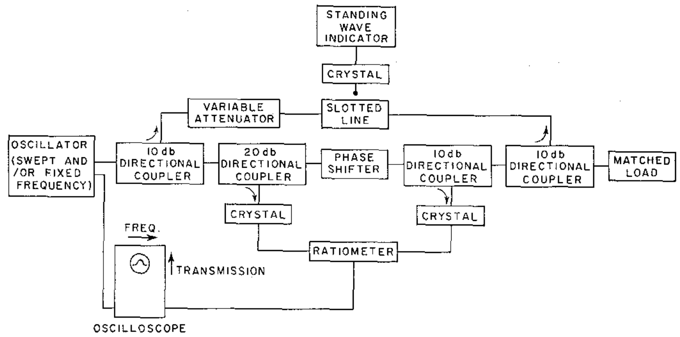

Fig. 5—Block diagram of experimental setup used to measure insertion loss and incremental phase shift.

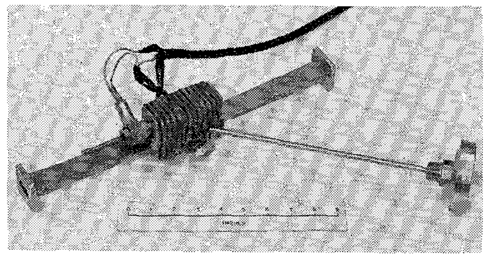

Fig. 6—Photograph of assembled phase shifter.

frequency. The dielectric constant and position of this half-wave line can then be determined for matching at f_2, the higher frequency.

The device illustrated in Fig. 3 was designed to operate between 8.7 and 9.3 kMc. Fig. 4 shows a comparison of the measured VSWR characteristics and the theoretical predictions. The results show that the measured VSWR is below 2.0 from 9.1 to 9.5 kMc over the complete bias range, giving the phase shifter a 400-Mc operating bandwidth. Generally, the curves tend to agree with the predicted behavior, indicating that this simple matching scheme is an effective one.

IV. Experimental Results

A block diagram of the experimental setup used to measure insertion loss and phase shift is shown in Fig. 5. Insertion loss is measured directly on the ratiometer while incremental phase shift is determined by measuring the change in minimum position on the slotted line due to a change in bias field and can be computed from

$$\Delta\phi = 2\beta\Delta z \text{ rad}, \tag{8}$$

where $\beta = 2\pi/\lambda_g$ is the propagation constant of the slotted line, and Δz is the shift in minimum position on the slotted line. The accuracy of these measurements is approximately ± 0.2 db and $\pm 1.0°$.

As shown in Figs. 1 and 2, the properties of the ferroelectric material are temperature sensitive and can be varied with a dc bias. The practical design of the electrically controlled ferroelectric phase shifter therefore requires a method for controlling the temperature and the bias field. The photograph of the assembled device shown in Fig. 6 illustrates how this is accomplished. To provide temperature stabilization for the ferroelectric ceramic a thermoswitch was mounted in an external metallic heat sink surrounding the waveguide and was connected in series with a Variac and lead covered heating cable wrapped around the outside of the heat sink. A thermometer was inserted into the heat sink to monitor the temperature. To keep the bias voltage below 3000 v, a 0.100-in waveguide height was used to house the ferroelectric and matching sections. To further reduce the voltage the ferroelectric slab was divided in half, and a biasing electrode was placed between the two half sections as illustrated in Fig. 7. A fine wire attached to the electrode was then brought out through the insulated high voltage probe shown in the center of the device in Fig. 6 in the manner indicated in Fig. 7. Arcing was prevented by introducing sulfur hexafluoride gas into the waveguide at atmospheric pressure.

Phase shift was measured in the 400-Mc pass band between 9.1 and 9.5 kMc. Fig. 8 shows the measured data together with the theoretical behavior at mid-band obtained from (7). The comparison was made at 66°C rather than 52°C used in the design, since it was found experimentally that the former was the optimum operating temperature. An examination of Figs. 1 and 2 indicates that the properties of the material are very similar in the temperature range between 52°C and 66°C. Since the dielectric constants and lengths of the impedance matching sections are only accurate to within certain tolerances, which vary for the different matching sections, one would expect the operating temperature of the device to differ somewhat from the design temperature in order to compensate for the discrepancies. The differences between theory and experiment shown in Fig. 8 can be accounted for by considering the effect of impedance mismatches at the various dielectric interfaces due to changes in the dielectric constant of the ferroelectric section. The theoretical curve determined from (7) implicitly assumes that the system is perfectly matched internally. Since this condition was not fully realized experimentally, the theoretical curve deviates slightly from the measured results.

The measured insertion loss of the phase shifter and

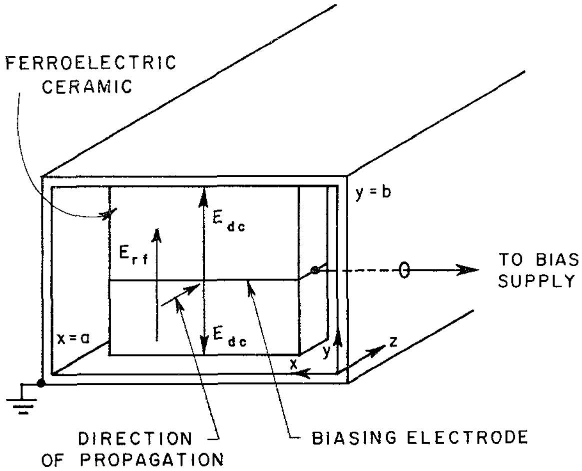

Fig. 7—Configuration of ferroelectric slab in waveguide showing biasing arrangement and representation of RF and dc fields.

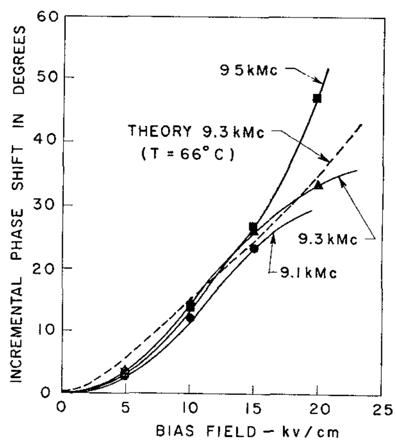

Fig. 8—Measured phase shift and theoretical prediction as a function of dc bias with frequency as a parameter.

the theoretical prediction, plotted as a function of frequency, are shown in Fig. 9. The theoretical curve was obtained by adding the insertion loss computed from (6) to the experimentally measured reflection loss determined from Fig. 4. The differences between the measured and the predicted behavior are explained by noting that the measurement of insertion loss gives the sum of the dissipation and the reflection loss of the entire device.[11] The dissipation losses in any network are always greater than the intrinsic losses since they take into account multiple reflections. In calculating the expected behavior of the phase shifter from (6), a fundamental assumption was that the ferroelectric slab was perfectly matched internally or that there were no internal resonances giving rise to dissipation loss. The difference between the measured and theoretical losses shown in Fig. 9 is therefore a measure of the losses due to internal resonances in the system. Since the loss tangents of the materials comprising the impedance matching sections are of the order of 10 per cent of the

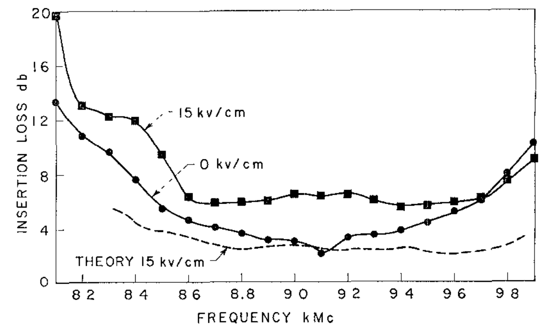

Fig. 9—Measured insertion loss and theoretical prediction as a function of frequency with bias as a parameter.

loss tangent of the ferroelectric, the over-all losses in the matching sections account for an appreciable fraction of the losses due to internal resonances.

V. Conclusions

The characteristics of the electrically controlled ferroelectric phase shifter shown in Figs. 4, 8, and 9 indicate that for the configuration given in Fig. 3 incremental phase shifts of 40° to 50° are attainable over reasonable bandwidths with insertion losses averaging about 5 db. These results are in general agreement with those obtained by other investigators working at much lower frequencies with an entirely different geometry.[12] The major disadvantages of the ferroelectric phase shifter are the necessity for temperature control in order to eliminate thermal fluctuations in the characteristics of the device, and the high values of insertion loss due to the losses in the dielectric materials. To estimate the magnitude of the variations in phase shift arising from thermal fluctuations, assume that the temperature of the ferroelectric ceramic varies by 5°C from the ambient temperature of 66°C. It is then found from (7) and the curves given in Fig. 1 that for a fixed bias of 10 kv/cm the phase shift can be expected to change by as much as 10° with accompanying small changes in the absorption and the input VSWR.

To determine the relative merit of the ferroelectric phase shifter, a comparison can be made with the behavior of phase shifters which employ other types of nonlinear elements. For example, Reggia and Spencer[13] have utilized ferrites to construct a reciprocal X-band phase shifter. Their findings show that with the proper configuration phase shifts in excess of several hundred degrees can be obtained with insertion losses in the neighborhood of 0.5 db for RF power levels up to about 25 w. Although these characteristics are superior to those obtained with the ferroelectric device, it should be noted that the ferrite phase shifter requires a longitudinal magnetic field for operation which is generated from a long solenoid. Consequently, the phase shifting control unit is bulky, limits the response time of the

[11] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 462–465; 1957.

[12] R. D. Hall, private communication, Sylvania Electric Products, Inc., Electron Defense Lab., Mountain View, Calif.; 1961.

[13] Reggia and Spencer, *op. cit.*, p. 1510.

device, and requires appreciable amounts of dc control power. To produce 40° to 50° of incremental phase shift in a ferrite device, approximately 50 mw of dc control power is required. To generate the same phase change in the ferroelectric device, the amount of dc control power needed is estimated to be about 100 μw which is considerably less than the dc powers necessary to drive the ferrite device. The construction of fast compact-diode phase shifters has been investigated by Hardin, Downey, and Munushian.[14] They have reported measuring phase changes of 41° at 9 kMc with an insertion loss of 3.9 db. These results are only valid for RF power levels in the vicinity of a few milliwatts due to the fact that the absorption increases with RF power in diodes. The amount of dc control power required to produce the measured phase change of 41° is very nearly the same as required by the ferroelectric device. From these results it can be seen that the electrically controlled ferroelectric phase shifter is comparable to the diode device in the X-band region. Furthermore, the ferroelectric phase shifter will operate satisfactorily as a CW device for RF powers at the watt level, which is similar to the power handling capabilities of the ferrite device and superior to those of the diode device.

Appendix

To determine the general incremental behavior of ferroelectric phase shifters consider the arbitrary waveguide configuration shown in Fig. 10, where reference planes 1 and 2 are chosen sufficiently far from the junction so that all higher order modes are vanishingly small at these points.

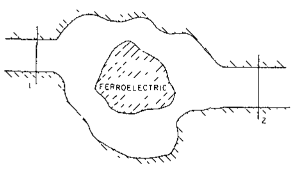

Fig. 10—Arbitrary waveguide configuration with ferroelectric.

It is desired to find the change in phase shift $\Delta\phi$ for a change in complex permittivity $\Delta\epsilon$ of the nonlinear ceramic. The system is assumed to be well matched so that reflection losses can be neglected. In addition wall losses are taken to be negligible in comparison to the losses occurring within the ferroelectric.

Within the volume bounded by reference planes 1 and 2 the fields must satisfy Maxwell's equations:

$$\begin{aligned} \nabla \times \boldsymbol{E} &= -j\omega\mu_0 \boldsymbol{H} \qquad & \nabla \times \boldsymbol{E}' &= -j\omega\mu_0 \boldsymbol{H}' \\ \nabla \times \boldsymbol{H} &= j\omega\epsilon \boldsymbol{E} \qquad & \nabla \times \boldsymbol{H}' &= j\omega(\epsilon + \Delta\epsilon)\boldsymbol{E}' \end{aligned} \tag{9}$$

where the primed quantities refer to the conditions existing after ϵ has been changed to $\epsilon+\Delta\epsilon$. Assume that the fields at reference plane 2 can be represented in terms of the fields at reference plane 1 by

$$\begin{aligned} \boldsymbol{E}_2 &= \boldsymbol{E}_1 e^{-\eta - j\phi} \qquad & \boldsymbol{E}_2' &= \boldsymbol{E}_1' e^{-\eta' - j\phi'} \\ \boldsymbol{H}_2 &= \boldsymbol{H}_1 e^{-\eta - j\phi} \qquad & \boldsymbol{H}_2' &= \boldsymbol{H}_1' e^{-\eta' - j\phi'} \end{aligned} \tag{10}$$

where

- ϕ is the total phase shift through the device, and
- η is the attenuation of the signal at reference plane 2 relative to reference plane 1.

For small changes in ϵ, one can write

$$\begin{aligned} \boldsymbol{E}' &= \boldsymbol{E} + \Delta\boldsymbol{E} \\ \boldsymbol{H}' &= \boldsymbol{H} + \Delta\boldsymbol{H} \end{aligned} \tag{11a}$$

and

$$\begin{aligned} \phi' &= \phi + \Delta\phi \\ \eta' &= \eta + \Delta\eta, \end{aligned} \tag{11b}$$

where only first order incremental changes are included.

The incremental phase shift $\Delta\phi$ can be computed by considering the vector identity

$$\begin{aligned} \nabla\cdot(\boldsymbol{E} \times \boldsymbol{H}'^* - \boldsymbol{E}' \times \boldsymbol{H}^*) &= \boldsymbol{H}'^*\cdot(\nabla \times \boldsymbol{E}) - \boldsymbol{H}^*\cdot(\nabla \times \boldsymbol{E}') \\ &\quad - \boldsymbol{E}\cdot(\nabla \times \boldsymbol{H}'^*) + \boldsymbol{E}'\cdot(\nabla \times \boldsymbol{H}^*), \end{aligned} \tag{12}$$

where * denotes the complex conjugate. Taking the volume integral of (12) over the volume V enclosed between reference planes 1 and 2 and converting the volume integral on the left hand side to a surface integral, one obtains the relation

$$\begin{aligned} \oint_S (\boldsymbol{E} \times \boldsymbol{H}'^* - \boldsymbol{E}' \times \boldsymbol{H}^*)\cdot d\boldsymbol{a} &= j\omega\mu_0 \int_V (\boldsymbol{H}'\cdot\boldsymbol{H}^* - \boldsymbol{H}\cdot\boldsymbol{H}'^*)dv \\ &\quad + j\omega\epsilon_0 \int_{V-V_f} (\boldsymbol{E}\cdot\boldsymbol{E}'^* - \boldsymbol{E}'\cdot\boldsymbol{E}^*)dv \\ &\quad + j\omega\epsilon^* \int_{V_f} (\boldsymbol{E}\cdot\boldsymbol{E}'^* - \boldsymbol{E}'\cdot\boldsymbol{E}^*)dv \\ &\quad + j\omega\Delta\epsilon^* \int_{V_f} \boldsymbol{E}\cdot\boldsymbol{E}'^* dv, \end{aligned} \tag{13}$$

where use has been made of (9). In deriving this equation, the volume integral containing terms in ϵ has been broken up into an integral over the volume of the ferroelectric V_f (the only place where the permittivity is complex and can undergo a change) and an integral over the remaining volume $V-V_f$ enclosed by the surface S. Due to the boundary conditions on the waveguide walls the surface integral vanishes everywhere except at the two reference planes S_1 snd S_2. For a constant input signal at reference plane 1 (*i.e.*, $\boldsymbol{E}_1' = \boldsymbol{E}_1$ and $\boldsymbol{H}_1' = \boldsymbol{H}_1$) it is found by substituting (10) and (11b) into (13) that

[14] Hardin, Downey, and Munushian, *op. cit.*, p. 944.

$$2j \sin \Delta\phi e^{-2\eta-\Delta\eta} \int_{S_2} (E_1 \times H_1^*) \cdot da$$
$$= -2\omega\mu_0 \int_V \mathrm{Im}\,(H' \cdot H^*) dv - 2\omega\epsilon_0 \int_{V-V_f} \mathrm{Im}\,(E \cdot E'^*) dv$$
$$- 2\omega\epsilon^* \int_{V_f} \mathrm{Im}\,(E \cdot E'^*) dv + j\omega\Delta\epsilon^* \int_{V_f} E \cdot E'^* dv, \quad (14)$$

where Im denotes the imaginary part. By equating the imaginary terms in (14) and assuming small losses (*i.e.*, $\epsilon'' \ll \epsilon'$) one obtains, to first order in Δ,

$$2\Delta\phi e^{-2\eta} \,\mathrm{Re}\left[\int_{S_2} (E_1 \times H_1^*) \cdot da\right] = \omega\Delta\epsilon' \int_{V_f} |E|^2 dv. \quad (15)$$

By making use of the relations for the output power P_{out} and the power lost in the dielectric P_{lost} given by

$$\tfrac{1}{2} e^{-2\eta} \,\mathrm{Re}\left[\int_{S_2} (E_1 \times H_1^*) \cdot da\right] = P_{\text{out}},$$

and

$$\tfrac{1}{2}\omega\epsilon' \tan\delta \int_{V_f} |E|^2 dv = P_{\text{lost}},$$

(15) reduces to

$$\Delta\phi = \frac{1}{2} \frac{\Delta\epsilon'}{\epsilon' \tan\delta} \frac{P_{\text{lost}}}{P_{\text{out}}}. \quad (16)$$

For small losses the insertion loss of the device is approximately given by

$$L \approx \frac{1}{0.23} \frac{P_{\text{lost}}}{P_{\text{out}}} \text{ db}. \quad (17)$$

Hence, by combining (16) and (17) and noting that $\epsilon' = \epsilon_0 \kappa'$, one obtains the formula for the incremental phase shift

$$\Delta\phi = 0.115 \frac{\Delta\kappa'}{\kappa' \tan\delta} L \text{ rad}. \quad (18)$$

A Balanced-Type Parametric Amplifier*

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Summary—A balanced-type diode amplifier is reported, in which the cutoff mode of the pumping waveguide resonating with the diode capacitances, is used as a signal circuit and a series connected diode loop is used as an idler. Theoretical noise-figure and gain-bandwidth product are derived after calculating the equivalent susceptance matrix of two diodes which are parallel-connected for the signal input and series-connected for the idler. This reveals that 1) the noise figure of the balanced-type amplifier can be expressed in the same form as that of the single diode amplifier, and 2) the gain-bandwidth product is identical to that of the single diode amplifier. In the experiment at 1900 Mc, a bandwidth of more than 200 Mc is obtained at the power gain of more than 10 db. A single-channel noise-figure of 2.5 db is measured at the pump power of 100 Mw.

Introduction

IN ORDER TO extend the bandwidth capabilities of parametric diode amplifiers, considerable attention is presently being given to traveling-wave devices. However, in the traveling-wave-type amplifiers,[1] the operation is almost limited to the degenerative case, except a few examples,[2] and their construction is still quite complicated in order to get good phase relations between signal and idler flows. A balanced-type diode parametric amplifier was, to the best of our knowledge, suggested by Takahashi[3] but had not been tested until Kliphuis[4] made a surprising success in obtaining a bandwidth of 500 Mc in his 5300-Mc experiment.

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